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$$=\frac{a-\frac{1}{a}}{2}\left[\frac{a+\frac{1}{a}}{a-\frac{1}{a}}-\frac{a^{2^n}+a^{-2^n}}{a^{2^n}-a^{-2^n}}\right].$$

When $a>1$, the limit of this sum is

$$\frac{a-\frac{1}{a}}{2}\left(\frac{a+\frac{1}{a}}{a-\frac{1}{a}}-1\right)=\frac{1}{a}.$$

Therefore, the sum of the series is the smaller root of the equation $x^2-a_1x+1=0$, viz., $\frac{a_1-\sqrt{(a_1^2-4)}}{2}$.

Note.—This furnishes a very rapid method for finding square roots to a considerable number of decimals. Example.—Let $a=16$,

$$8-3\sqrt{7}=\frac{1}{16}+\frac{1}{16.254}+\frac{1}{16.254.645.64514}+\ldots$$

and three terms of the series give $\sqrt{7}$ to 18 decimals, four terms will give 37 decimals, etc.

Also solved by S. Lefsehetz.

GEOMETRY.

365. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Given the coordinates of the four vertices of the tetrahedron, (x_1, y_1, z_1); (x_2, y_2, z_2); (x_3, y_3, z_3); (x_4, y_4, z_4): find volume and express it by a determinant.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and I. W. SMITH, A. M., Assistant Professor of Mathematics, North Dakota Agricultural College.

Let Δ =area of triangular base *BCD* of the tetrahedron, p the perpendicular from the vertex *A* on the base *BCD*, *V* its volume, $(x-x_2)\cos\alpha+(y-y_2)\cos\beta+(z-z_2)\cos\gamma=0$ the equation to the plane of *BCD*.

Then $\Delta\cos\gamma$ is the projection of the area *BCD* on the plane xy, and (x_2, y_2), (x_3, y_3), (x_4, y_4) are its angular points.

$$\therefore 2\triangle\cos\gamma=\begin{vmatrix}1, & 1, & 1\\ x_2, & x_3, & x_4\\ y_2, & y_3, & y_4\end{vmatrix}=\begin{vmatrix}x_3-x_2, & x_4-x_2\\ y_3-y_2, & y_4-y_2\end{vmatrix}.$$

Similarly,

$$2\triangle\cos\beta=\begin{vmatrix}z_3-z_2, & z_4-z_2\\ x_3-x_2, & x_4-x_2\end{vmatrix},\quad 2\triangle\cos\alpha=\begin{vmatrix}y_3-y_2, & y_4-y_2\\ z_3-z_2, & z_4-z_2\end{vmatrix}.$$

Also, $-p=(x_1-x_2)\cos\alpha+(y_1-y_2)\cos\beta+(z_1-z_2)\cos\gamma$.

$$\therefore -6V=-2\triangle p=2\triangle\cos\alpha(x_1-x_2)+2\triangle\cos\beta(y_1-y_2)+2\triangle\cos\gamma(z_1-z_2)$$

$$=(x_1-x_2)\begin{vmatrix}y_3-y_2, & y_4-y_2\\ z_3-z_2, & z_4-z_2\end{vmatrix}+(y_1-y_2)\begin{vmatrix}z_3-z_2, & z_4-z_2\\ x_3-x_2, & x_4-x_2\end{vmatrix}$$

$$+(z_1-z_2)\begin{vmatrix}x_3-x_2, & x_4-x_2\\ y_3-y_2, & y_4-y_2\end{vmatrix}$$

$$=\begin{vmatrix}x_1-x_2, & x_3-x_2, & x_4-x_2\\ y_1-y_2, & y_3-y_2, & y_4-y_2\\ z_1-z_2, & z_3-z_2, & z_4-z_2\end{vmatrix}=-\begin{vmatrix}1, & 1, & 1, & 1\\ x_1-x_2, & 0, & x_3-x_2, & x_4-x_2\\ y_1-y_2, & 0, & y_3-y_2, & y_4-y_2\\ z_1-z_2, & 0, & z_3-z_2, & z_4-z_2\end{vmatrix}$$

Multiply the first row of this last determinant by x_2, y_2, z_2 and add to the second, third, fourth, respectively, we get

$$V=\tfrac{1}{6}\begin{vmatrix}1, & 1, & 1, & 1\\ x_1, & x_2,, & x_3, & x_4\\ y_1, & y_2,, & y_3, & y_4\\ z_1, & z_2,, & z_3, & z_4\end{vmatrix}$$

Also solved by S. G. Barton, and V. M. Spunar.

366. Proposed by G. I. HOPKINS, A. M., Professor of Mathematics and Astronomy, Manchester, N. H.

Construct a triangle, having given the base, vertical angle, and difference of altitude and difference of other two sides.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Mo.

Let A be the given vertical angle, a the given side, and d=given difference. Then $h-(b-c)=d$, or $b-c=h-d$... (1).

We have also $b^2+c^2-2bc\cos A=a^2$, which by means of (1) changes into $4bc\sin^2\frac{1}{2}A=a^2-(h-d)^2$, whence $bc=\dfrac{a^2-(h-d)^2}{4\sin^2\frac{1}{2}a}$; but $bc\sin A=ah$.

$\therefore\ \dfrac{a^2-(h-d)^2}{4\sin^2\frac{1}{2}a}=\dfrac{an}{\sin A}$, and thus we finally get the quadratic equa-